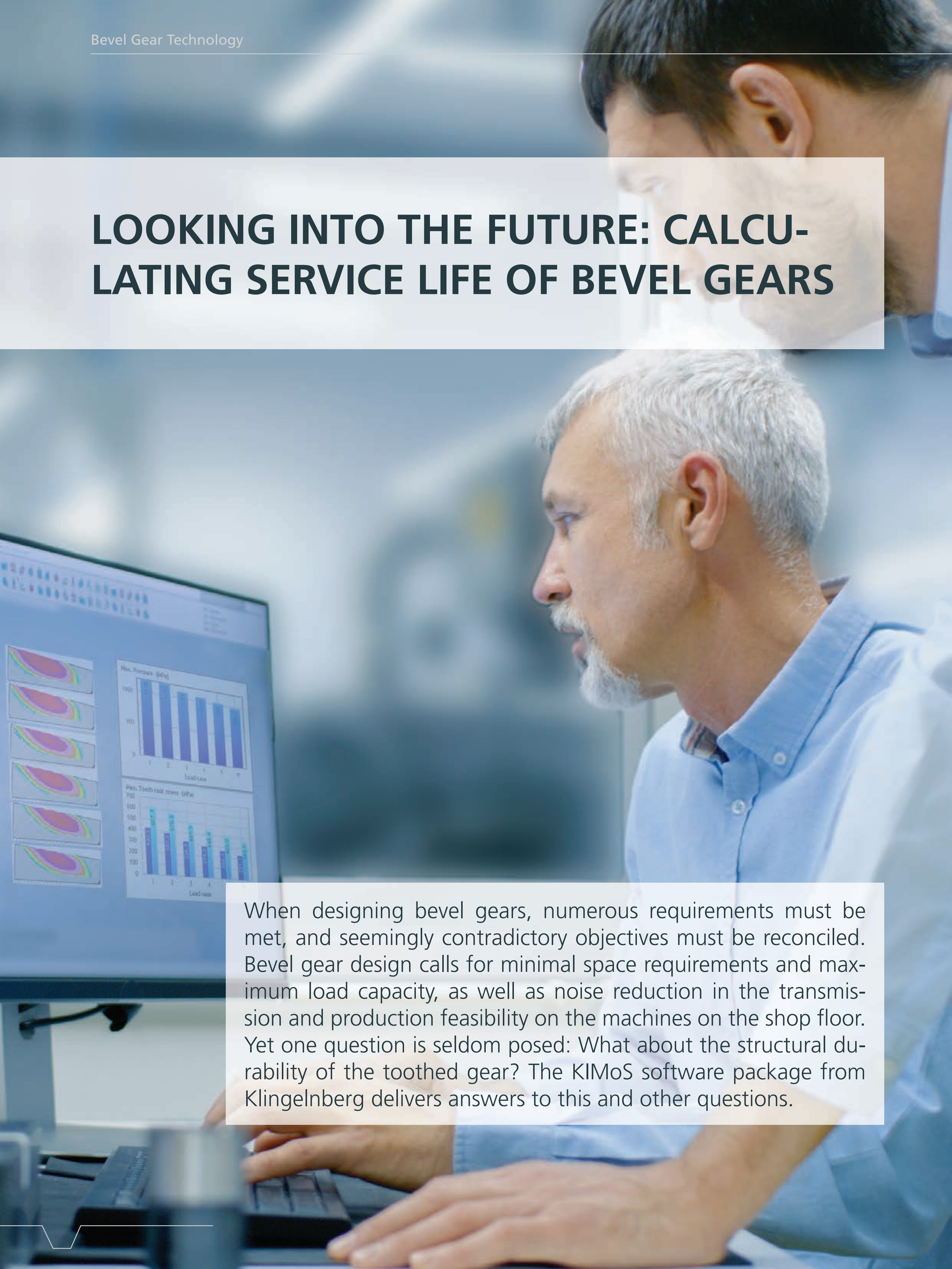


LOOKING INTO THE FUTURE: CALCULATING SERVICE LIFE OF BEVEL GEARS

When designing bevel gears, numerous requirements must be met, and seemingly contradictory objectives must be reconciled. Bevel gear design calls for minimal space requirements and maximum load capacity, as well as noise reduction in the transmission and production feasibility on the machines on the shop floor. Yet one question is seldom posed: What about the structural durability of the toothed gear? The KIMoS software package from Klingelnberg delivers answers to this and other questions.

If the maximum load applied to a gear tooth does not exceed the load limits of the material, the tooth will return to its initial condition after the load is removed. This assumption holds true for several hundred load applications. But when we are dealing with several million loadings, gear tooth damage occurs far below the static load limits of the material. This phenomenon is known as fatigue.

A broad range of gear damage is documented in the literature. These damage types are classified and illustrated in the book titled Maschinenelemente[1] [Machine Elements] by Gustav Niemann and Hans Winter. Figure 1 shows the boundary lines at which each type of gear damage occurs as a function of the sliding velocity and torque applied. The position of the boundary lines for wear, micro-pitting, and scuffing can be further displaced by the roughness parameters of the tooth flank, as well as the lubrication conditions and addendum modification. It is difficult, however, to obtain statistically reliable findings for a service life calculation.

Root Cause Analysis

Calculating a material's service life requires an examination of the tooth damage that can actually be attributed to material fatigue. Scuffing damage, for example, is caused by insufficient lubrication between the tooth flanks rather than material fatigue. In the case of flank ruptures, an uncertain picture emerges. Tooth flank fracture is a fatigue damage that occurs without a clear explanation. It is currently assumed that there is a small material defect at a location beneath the surface where the stress attains a maximum level, and that this defect is the point at which an initial crack forms. This crack then progresses until it reaches the opposite side of the tooth, and the tooth breaks away in a sickle-like formation. Effects due to material purity and heat treatment, which cannot be measured statistically, make it difficult to calculate the service life of a gear for tooth flank fracture.

Tooth Fractures and Pitting Damage

Tooth fractures and pitting damage, as depicted in figures 2 and 3, are typical manifestations of material fatigue-induced gear damage. When a tooth fractures, the local tooth root stress exceeds the endurance strength of the material. The maximum tooth root stress lies near the 30-degree tangent of the root fillet. A small initial crack in this location continues to develop, until the entire tooth ultimately breaks away from the gear body. This is clearly evident in figure 2.

Pitting damage is caused by stress on the flanks in the tooth contact. This contact stress fatigues the hard surface of the tooth flank. Micro-cracks develop, into which lubrication is compressed during roll-over. The sudden increase in pressure causes small particles to break off. The further this damage progresses, the more running behavior of the gearing deteriorates, due to the incremental disruption of an smooth tooth contact. Figure 3 shows this type of pitting damage. The transmission is still functional at the level of damage depicted here. As the damage increases, the entire tooth is ultimately affected by the chipping. Due to the absence of a case hardening surface layer and the interruption of the lubricating film, this in turn results in scuffing and tooth fracture.

Determining the Tooth Root Load Capacity

So how does one ascertain the material fatigue-limited mechanical strength

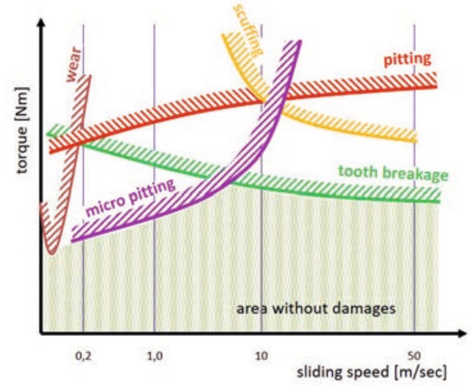


Fig. 1: Possible toothed gear damage

Fig. 2: Tooth fracture damage

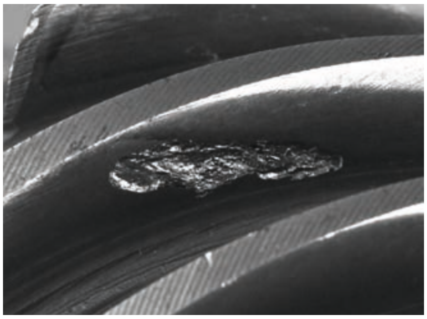


Fig. 3: Pitting damage

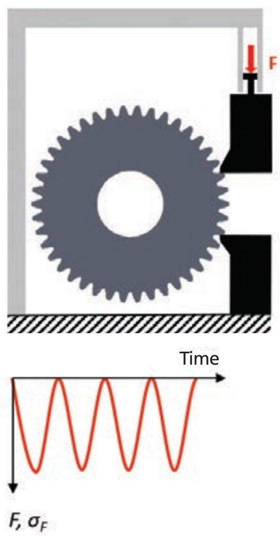


Fig. 4: Pulsator test stand

properties of a gear material? This is done by performing practical tests, which must then be verified statistically. One of these tests involves determining the tooth root load capacity. Figure 4 provides a schematic view of a pulsator test stand. A gear is clamped between two jaws on a typically hydraulic test stand. One of these jaws is fixed and measures the force applied using a load cell. The other jaw executes a pulsating motion, thereby exposing the tooth root to an alternating stress cycle. Using a simulation program, the tooth root stress is calculated based on the introduced force.

These pulsator tests are conducted at many different force levels and always to the point of tooth fracture. The result is interesting: At extremely high root stresses near the static load limit of the material, the tooth fractures after a few stress cycles. The lower the root stresses, the more stress cycles the gear withstands until the tooth fractures. If the root stress remains below a certain level, tooth fracture does not occur even after several million stress cycles.

The basic principles and evaluations of these experiments can be traced back to August Wöhler, who investigated the cause of fractures in railroad axles in the nineteenth century. Figure 5 shows the result of a long test series on the pulsator test stand. Every point in red or green is a test sequence. The points in red stand for an end of testing due to tooth fracture, and the points in green stand for an end after achieving a certain number of stress cycles without a tooth fracture. The points for a given load level are distributed more or less broadly depending on the purity of the material and the quality of the heat treatment.

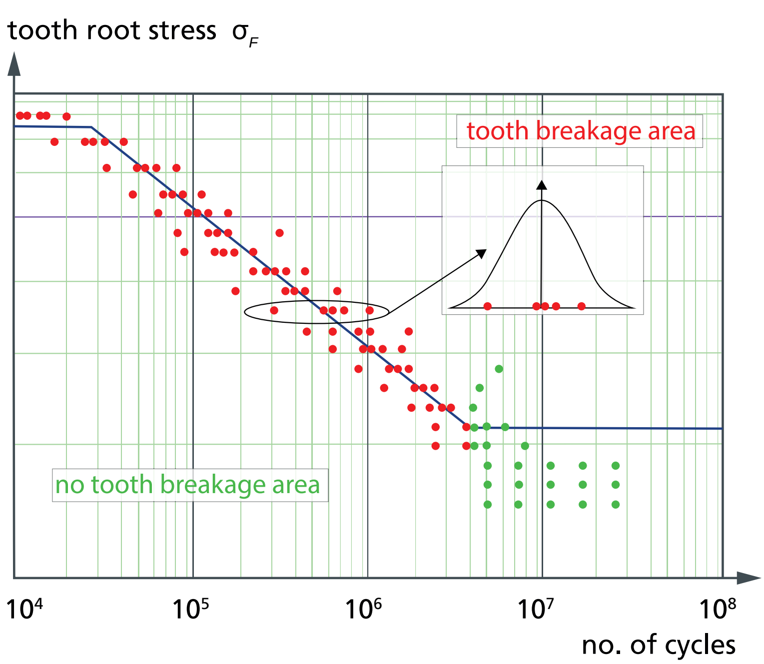


Fig. 5: Results of a tooth fracture test series on the pulsator test stand

Short-Term Strength, Fatigue Strength, and Endurance Strength: the Wöhler Curve

In the log-log plot depicted in figure 5, three areas of the S-N curve, also known as a Wöhler curve, can be distinguished. The first area is the short-term strength area, where the gear withstands extremely high stresses but only few stress cycles. The adjoining area is the fatigue strength area, in which the number of stress cycles that can be withstood increases as the load decreases. If no damage occurs after an extremely high number of stress cycles, this is referred to as endurance strength, the third part of the Wöhler curve. This three-part division of the Wöhler curve, which is statistically verified only, leads to the following formula in the fatigue strength area:

$$N = N_{\lim} \cdot \left(\frac{\sigma_{\lim}}{\sigma}\right)^q \quad \Leftrightarrow \quad \log\frac{N}{N_{\lim}} = q\log\left(\frac{\sigma_{\lim}}{\sigma}\right)$$

One observes that when expressing the equation as a logarithm, the stress σ against the number of stress cycles N forms a straight line in a log-log coordinate system. The values σ_{lim} and N_{lim} correspond to the transition on the Wöhler curve from fatigue strength to endurance strength. Under this assumption, the Wöhler curve parameters q, σ_{lim} and N_{lim} can be determined with just a few pulsator tests.

Wöhler Curve for Pitting Damage

Failure behavior prediction can be used for all types of damage in gears that have an endurance strength area. In addition to tooth fractures, a Wöhler curve can also be plotted for pitting damage. The test stand setup outlined in figure 6 is usually a gear test stand in which two cross-located gears are rotating. The imposed torque is converted to Hertzian contact stress using a computation program. The test sequence is very similar to the pulsator test described above. In this case, however, the test is ended when the pits reach a certain size rather than upon complete failure of the gear pair.

The results can also be plotted in a Wöhler curve (see figure 7). Here too, the Wöhler curve parameters for the fatigue strength area can be determined with a few test points. Thus the time it takes for a toothed gear to sustain a tooth root fracture or pitting damage at a predefined load can be calculated specifically.

In real-world applications, however, the loads occurring in a transmission vary over time. In stationary transmissions, the number of load cases may be extremely limited, but vehicle transmissions in particular are subjected to many different loading conditions.

Failure behavior prediction can be used for all types of damage in toothed gears that have an endurance strength area.

There are various counting methods used to determine a discrete load spectrum for a gear pairing based on the time profile of the torque, such as the rainflow method, which transforms time-dependent cyclic loads into discrete load spectra.

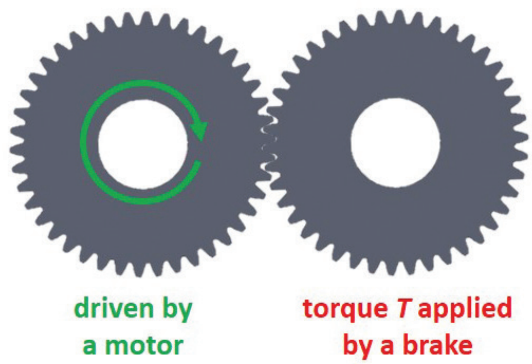


Fig. 6: Rotation of two cross-located toothed gears

Cumulative Damage Hypothesis for Service Life Calculation

So how can the service life of a gear pair be calculated using a Wöhler curve and an existing load spectrum? Miner and Palmgren's cumulative damage hypothesis provides a solution.

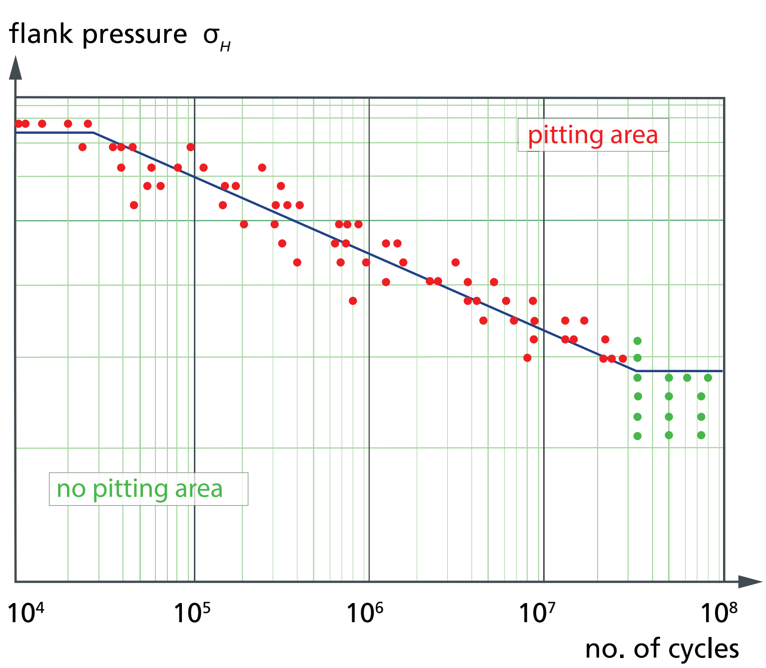


Fig. 7: Result of a pitting test series on the pulsator test stand

Figure 8 shows a load with three load levels. Each load case i in the load spectrum is described by its number of stress cycles N_i and its associated stress σ_i. The number of maximum sustainable stress cycles N_{fi} for stress σ_i is taken from the Wöhler curve, and a partial damage value is calculated $D_i = N_i / N_{fi}$. Because the load in load case 3 falls within the endurance strength area, there is no partial damage for this load case. All occurrences of partial damage are added together to obtain the total cumulative damage D. If the total damage D exceeds 1, damage will likely occur. If D is less than 1, damage will not likely occur. The certainty of this conclusion depends on the extent to which the real material can be adequately described by the Wöhler curve. If the material properties are widely distributed, this conclusion has a very low certainty.

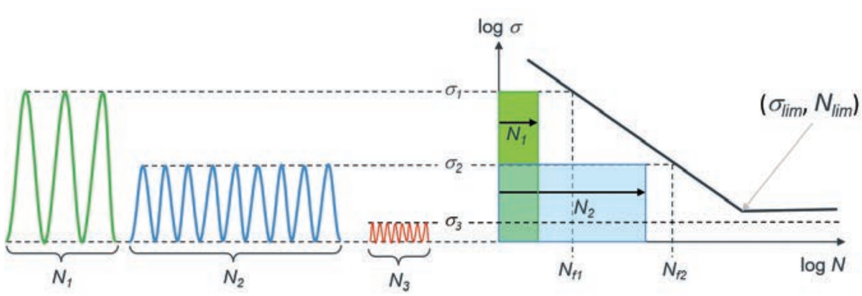


Fig. 8: Load cases and damage fractions

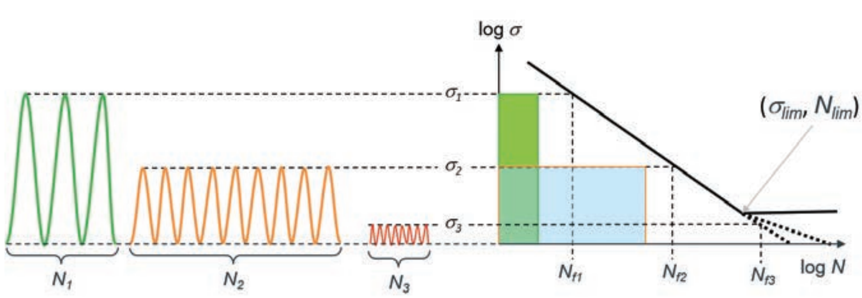


Fig. 9: Load case below endurance strength with damage fraction

Signs of Fatigue – Even in the Endurance Strength Area

According to this formula, all load applications with associated stresses below σ_{lim} should have no effect on the total damage D. Unfortunately, practical experience with gear damage reveals another behavior. It seems that load applications below the endurance strength area combined with higher stresses also induce signs of fatigue in the material. In order to accommodate this phenomenon, the third section of the Wöhler curve has been modified to computationally approximate cases occurring in the real world. Instead of a horizontal line starting from the point (σ_{lim}, N_{lim}), the Wöhler curve now follows a sloping path, as shown in figure 9. The elementary form of the Miner rule uses the slope coefficient q' = q for this. The Miner rule modified by Haibach uses the slope coefficient q'= (2q-1).

It is evident from these modifications – as empirical as they are – that the structural durability calculation still cannot predict the service life of a gear pair with sufficient accuracy. In any case, a conservative estimate can be obtained using the Miner elementary approach.

Determining Service Life

The service life can now be calculated based on the findings for the total damage D. Empirical values play a key role here as well. It is assumed that the end of the service life is attained at a total damage $D_{0.85}$ = 0.85 for pitting and $D_{0.3}$ = 0.3 for tooth fracture. The number of stress cycles can be calculated based on the rotation speeds of the load spectrum levels and the corresponding service life. The stress cycles N_{life} permitted until total damage $D_{0.85}$ are then obtained from $N_{life} = (\sum N_i) \cdot D_{0.85} / D$. The service life in hours can now be easily calculated using the rotation speed of the gear.

Special Characteristics of Bevel Gears

With all of the many assumptions and approximations in the Wöhler curves and the cumulative damage hypothesis, an additional effect must be taken into account when dealing with bevel gear sets. Here, every load level exhibits a different load-induced displacement of the pinion with respect to the ring gear. These displacements are indicated in the direction of the axis offset, in the axial directions of the pinion and ring gear axis, and as a deviation from the shaft angle. Different flank areas are stressed for each load case, depending on the ease off of the gear set. In order to take this effect, which is typical for bevel gears, into account, it is not sufficient to observe the maximum values of the stress independently of position. Instead, a local approach on the flank is called for.

The contact stress distribution for six load cases is shown on the left in figure 10. The tooth flank, consisting of individual tiles, can be seen on the right. For the first load case, the contact stress distribution occurs on the individual tiles. Thus an individual contact stress is applied to each tile, resulting in individual partial damage. The total damage of a tile, then, is the sum of the partial damage resulting from each individual load case.

Ease-Off Topography for Service Life Calculation

The service life calculation for tooth fracture and pitting is demonstrated below using two different ease-off topographies as an example.

The associated load contact patterns on the ring gear flank and the values of the Hertzian contact stress and the tooth root stress are shown in figure 12. Load case 1 exhibits the greatest contact stress at 1,198 MPa. In looking at the load contact patterns, it is evident that the contact pattern shifts in the heel-to-tip direction on the ring gear as the torque increases.

Compact

KIMoS

The KIMoS (Klingelnberg Integrated Manufacturing of Spiral Bevel Gears) software package supports every step in bevel gear design and optimization. KIMoS enables fast, accurate analysis of testing and production results, as well as gear damage.

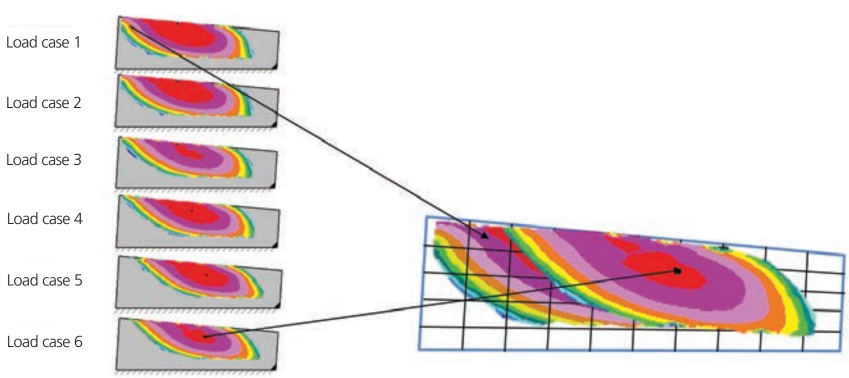
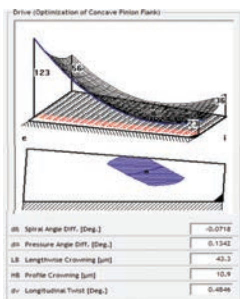


Fig. 10: Local determination of damage value

SIDE-BY-SIDE COMPARISON OF TWO EASE-OFF TOPOGRAPHIES WITH DIFFERENT DISPLACEMENT CHARACTERISTICS

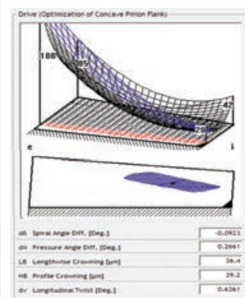


Toothing data

Number of teeth	13/38
Ring gear reference circle diameter	230 mm
Axis offset	20 mm
Face width	35 mm

Load case	Hours [h]	Rotation Speed [rpm]	Torque [Nm]	Displacement V/H [mm]
1	1,000	1,500	600	-0.25 / +0.20
2	2,000	500	550	-0.22 / +0.19
3	3,000	900	450	-0.17 / +0.15
4	1,000	3,600	400	-0.15 / +0.13
5	2,000	2,400	350	-0.10 / +0.08
6	1,000	1,800	300	-0.08 / +0.06

Fig. 11: Standard ease-off and load spectrum



Toothing data

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Load case	Hours [h]	Rotation Speed [rpm]	Torque [Nm]	Displacement V/H [mm]
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2	2,000	500	550	-0.22 / +0.19
3	3,000	900	450	-0.17 / +0.15
4	1,000	3,600	400	-0.15 / +0.13
5	2,000	2,400	350	-0.10 / +0.08
6	1,000	1,800	300	-0.08 / +0.06

Fig. 14: Optimized ease-off and load spectrum

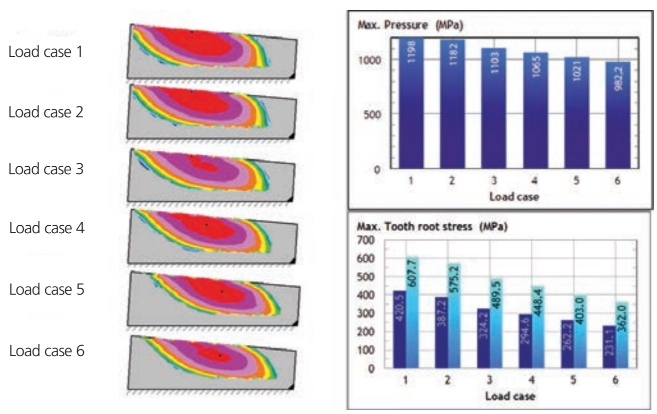

Fig. 12: Load contact patterns and stresses for standard ease-off

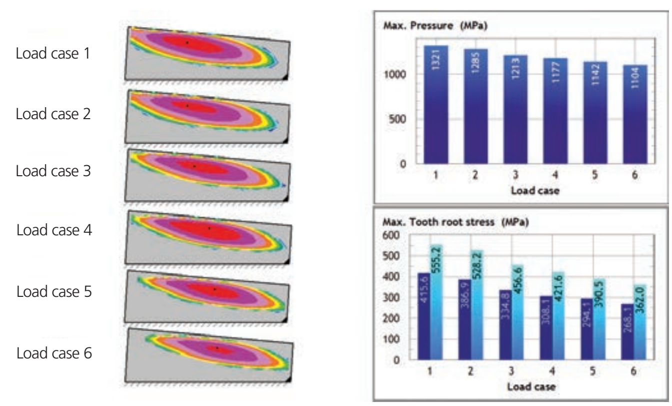

Fig. 15: Load contact patterns and stresses for optimized ease-off

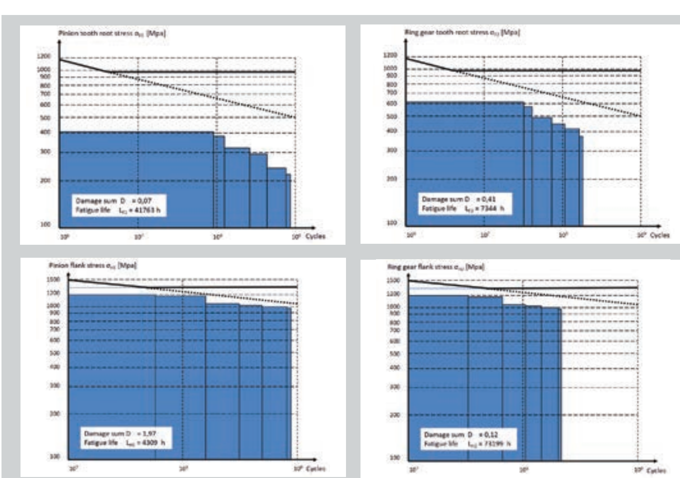

Fig. 13: Service life calculation with standard ease-off

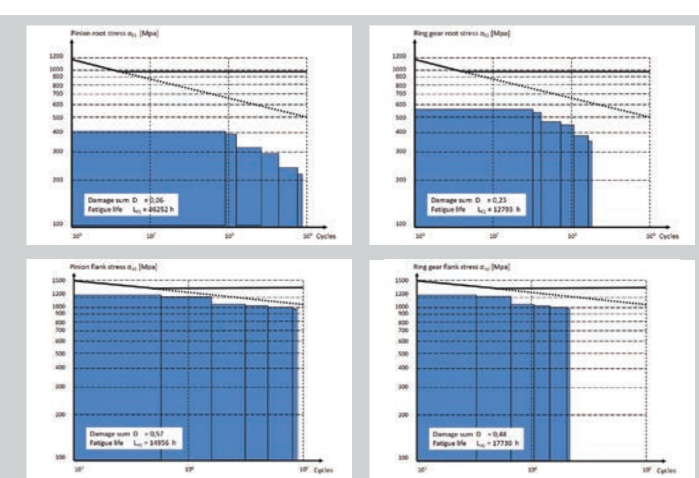

Fig. 16: Service life calculation with optimized ease-off

The greatest tooth root stress also occurs in load case 1, where 607.7 MPa is applied to the ring gear root.

Figure 13 shows the service lives identified. The total damage on the tooth root is 0.07 for the pinion and 0.41 for the ring gear. This ratio likewise corresponds to the strongly divergent tooth root stresses. The total damage on the pinion flank is 1.97 and will very likely result in pitting damage on the pinion. Thus the total service life in the load spectrum of 10,000 hours will not be achieved, and the predicted total service life is 4,309 hours.

Modifying the ease-off topography such that the displacement behavior changes will also significantly increase the service life. Figure 14 shows the same gearing with a different ease-off. The load spectrum and the displacements are the same.

Figure 15 shows the load contact patterns. The other displacement behavior is clearly apparent. The contact pattern shift is more pronounced, and more tiles are loaded overall as a result. The maximum Hertzian contact stress of 1,321 MPa for load case 1 is ten percent above the other example.

Nevertheless, a longer service life can be expected with regard to pitting on the pinion. This is also confirmed in the calculation. Instead of the original 4,309 hours, the calculation here shows a value of 12,128 hours with a total damage of 0.70. Compliance with the required operating duration has been achieved by changing the microgeometry, without increasing the installation space or costs.

Key patterns can be identified using the service life prediction feature for bevel gear sets implemented in KIMoS.

Conclusion

Although Wöhler curves provide only a limited characterization of the actual material behavior and the Miner-Palmgren cumulative damage hypothesis provides an extremely conservative estimate, key patterns in a bevel gear set can be identified using the service life prediction feature implemented in KIMoS (Klingelnberg Integrated Manufacturing of Spiral Bevel Gears). The local approach, in particular, which determines damage locally on the tooth flank, is an important calculation method for real-world applications due to the characteristic displacement behavior of bevel gear systems. ◆

Literature:
[1] Gustav Niemann, Hans Winter, Maschinenelemente Band 2 [Machine Elements, Volume 2], Springer Verlag, Berlin, 1986

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